A note on the bathtub vortex

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(Received 26 March 1962)

Observations were made of the vortex motion which occurs when a liquid is draining from a vessel through a hole in its bottom. It was found that, for relatively quiescent initial conditions, the direction of rotation *reversed* as the liquid surface approached the bottom of the vessel. An explanation of this phenomenon is proposed which is based upon the vorticity associated with the circumferential component of the flow in the boundary layer on the bottom of the vessel.

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1. Introduction

The swirling motion which occurs while the water in a bathtub is being emptied is a familiar occurrence. Thus the term 'bathtub vortex' is commonly used to refer to the vortical motion which generally occurs while a liquid-containing vessel is being emptied under the influence of gravity through a hole in its bottom. The standard explanation of the phenomenon, which implicitly assumes that the fluid is inviscid, applies the principle of conservation of angular momentum and leads to the conclusion that 'In all such cases the moving fluid must previously have possessed circulation' (Prandtl 1949, p. 69). On the other hand, a recent experiment by Weske (1958) has shown that under some conditions the combination of a rectilinear boundary-layer flow with a symmetrical sink flow is unstable in the sense that amplification of perturbations of the secondary vorticity associated with the curved streamline pattern led to the creation of circulation around the sink outlet. This observation led Weske to suggest that the circulation in the bathtub vortex might be created in a similar manner. The experiments reported here were originally undertaken to test this alternative to the standard, inviscid explanation of the origin of the bathtub vortex.

2. Experiments

2.1. *Apparatus*

A cylindrical 'bathtub' (shown in figure 1) having a diameter of 12 in. and a flat bottom was constructed of transparent Lucite to facilitate dye observations. The flow outlet at the centre of the tub had an inside diameter of 0·180 in. and varied in height b from a sharp edged orifice to $b = 1$ in. The initial height of the water z_0 varied from 0·5 to 4 in. The tub was filled at a standardized rate through a filler tube fixed at an angle of $\pm 20^\circ$ to give initially clockwise or counter-clockwise circulation. The residual circulation at the time a run was started by removing the outlet plug, $t = 0$, was controlled by varying the settling period between the completion of filling and the start of the run.

2.2. *The origin of the vortex*

Observations using powder and dye techniques quickly showed that for settling periods of a few hours or less (i) the direction of rotation of the vortex coincided with the direction in which the tub was filled, and (ii) the strength of the vortex decreased as the settling period increased, that is, as the residual circulation decreased. While these results were being obtained, the author learned that Shapiro using a six-foot-diameter tub and settling periods of several days had obtained a consistently counter-clockwise direction of rotation independent of the direction in which the tub was filled,† a result attributable to the action of the Coriolis force in the Northern hemisphere.

These results combine to support the standard, inviscid explanation of the origin of the bathtub vortex as opposed to Weske's hypothesis.

2.3. *Vortex reversal*

As the tests proceeded, the author frequently observed that near the end of a run the direction of rotation of the vortex reversed. This entirely unexpected result became the main object of the investigation, and the following technique was developed to record the phenomenon.

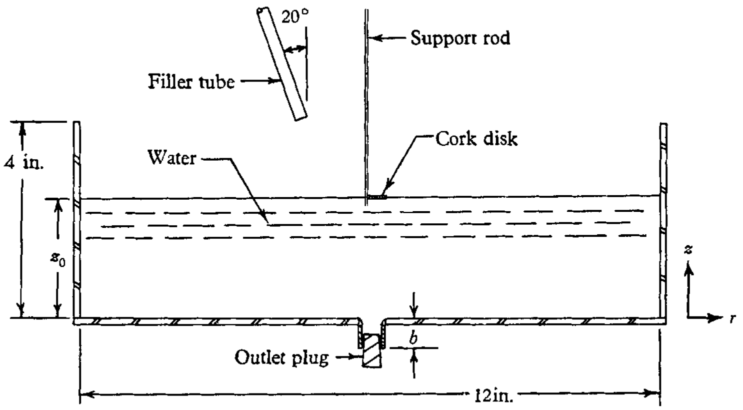


FIGURE 1. Apparatus used for observing the bathtub vortex.

A thin cork disk having a diameter of 4 mm and a weight of 0·8 mg was kept floating at the centre of the tub during the settling period by means of the support rod shown in figure 1, since surface tension causes the disk to be attracted to the rod. When the surface of the water drops below the support rod, this constraint on the motion of the disk is removed and the disk tends to centre itself on the axis of the vortex. Motion pictures of the rotating disk were

† This is demonstrated in the film 'Vorticity' available from Educational Services Incorporated, 47 Galen St., Watertown, Massachusetts. The experiment was done under the direction of Prof. A. H. Shapiro at the Massachusetts Institute of Technology. The Coriolis force would determine the direction of the vortex when the residual circulation relative to the tub is less than the circulation due to the normal component of the earth's rotation which varies from zero at the equator to one revolution per day (0·0007 rpm) at the poles.

analyzed to obtain its angular displacement and angular velocity; a typical result is shown in figure 2.

Vortex reversal was observed for either initial direction of rotation and over the range of initial water height z_0 and outlet length b given in §2.1. Reversal almost always occurred when the water surface z_s was between 0·4 and 0·2 in. The main factor affecting reversal was the settling time, the optimum values of which were between 5 and 20 min for this apparatus. For shorter settling times reversal did not usually occur, while for longer times the vortex was so weak as to make observation difficult. The instantaneous efflux velocity varies as $(z_s+b)^{\frac{1}{2}}$, and the main effect of increasing b for a fixed z_0 was to decrease both the total efflux time and the proportion of the total time during which the direction of rotation was reversed.

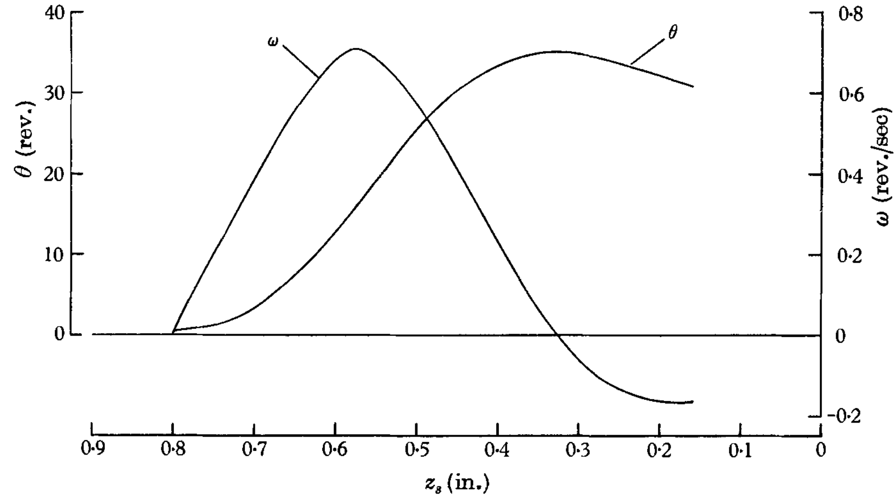


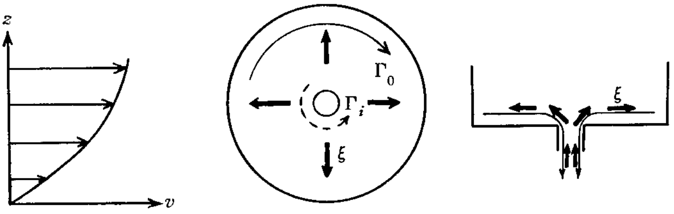
FIGURE 2. Angular displacement θ and angular velocity ω at the water surface on the axis of the vortex as a function of surface height z_s; $z_0 = 0{\cdot}9$ in., $b = 0{\cdot}5$ in. The disk floated free of the rod at $z_s = 0{\cdot}8$ in., and the water surface broke at $z_s = 0{\cdot}16$ in. at $t = 146$ sec.

3. Proposed explanation of reversal

As the liquid drains out of the tub, a boundary layer develops on its bottom. Away from the outlet, the velocity in this boundary layer may be divided into radial and circumferential components; the circumferential velocity profile $v(z)$ is indicated in figure 3(*a*). This velocity distribution gives rise to a radial component of vorticity ξ. If the circulation at the beginning of the run Γ_0 is in the clockwise direction and a right-hand convention is used, the vorticity vector ξ will be in the direction shown in figure 3(*b*).

For inviscid flow, a well-known theorem of Helmholtz states that a vortex line always consists of the same fluid particles and therefore moves with the fluid, and this result may usefully be applied to fluids of small viscosity such as water. When the fluid in the boundary layer approaches the outlet it acquires a vertical component of velocity, and, as a consequence of Helmholtz's theorem, the vorticity vector ξ also acquires a vertical component as indicated in figure 3(*c*).

Associated with this vertical component of ξ will be an induced circulation Γ_i which is *opposite* in direction to Γ_0; the strength of Γ_i will increase as the proportion of fluid in the exit stream which has come from the boundary layer increases. Just such an increase in the proportion of boundary-layer fluid in the exit stream occurs as the water surface approaches the bottom of the tub, and it is suggested that at this time the induced circulation Γ_i becomes sufficiently strong to account for the observed reversal in the direction of rotation.



(*a*) (*b*) (*c*)

FIGURE 3. The effect of the boundary layer on the flow out of the tub.

These heuristic arguments are of course not considered to be conclusive, and alternative explanations of this phenomenon may be proposed.

The assistance of N. Fong in performing these experiments is gratefully acknowledged.

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WESKE, J. R. 1958 On the origin and mechanism of vortex motion at the inlet of intakes placed near a flat surface. *University of Maryland, Tech. Note* BN-152, AFOSR-TN-58 863.